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Further Evidence for Magnetic Charge from Hadronic Spectra

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Schwinger's dyonium model of quarks is incorporated into a theory of the strong QCD and electromagnetic interactions. Vector bosons are shown to exist with masses obeying Nambu's empirical mass formula $m_n = (n/2)137m_e$, n a positive integer. The existence of a Dirac unit of magnetic charge $g = (137/2)ne$ is the basis for the Nambu formula and for the 70-MeV quantum in MacGregor's constituent-quark model. Frosch derived an empirical mass formula which contains the Nambu mass series as a subset.

1. INTRODUCTION

Since the conjecture by Dirac (1931) that magnetic monopoles might exist, it has been speculated by Schwinger (1969) and Chang (1972) that quarks consist of electric and magnetic charges. In non-Abelian gauge theories, the confinement of quarks is connected with monopoles (Mandelstam, 1975; Nambu, 1974). If magnetic monopoles exist, they are inseparably linked with massive vector bosons as suggested by 't Hooft (1974) and Polyakov (1974).

In the present paper, we investigate the role dyonic quarks play in the origin of elementary particle mass. In Section 2, Dirac magnetic monopoles are treated as Goldstone and Higgs bosons within the framework of the theory of the strong QCD and electromagnetic interactions. Nambu's empirical mass formula (Nambu, 1952) is shown to be based upon the existence of the Dirac magnetic charge. In Section 3, we discuss how Benjwal and Joshi (1987) derived similar expressions for particle masses. From the constituent-quark model of MacGregor (1990), the experimental data support the existence of a 70-MeV mass quantum and of magnetic

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charge $g = (137/2)ne$. A comparison of Nambu's formula and MacGregor's model is presented in Section 4. In Section 5, we show that Frosch's derivation of the empirical mass formula $m \sim N(3m_e)$ has as its basis the existence of a Dirac unit of magnetic charge. Finally, concluding remarks are made in Section 6.

2. DIRAC MAGNETIC MONOPOLES AS GOLDSTONE AND HIGGS BOSONS

We consider the problem of incorporating Schwinger's dyonium model of quarks into a theory of the strong QCD and electromagnetic interactions. The procedure is to first write down a modified QCD Lagrangian in which massless Yang–Mills fields $\mathbf{G}^{a\mu}$ interact with a multiplet of scalar Higgs fields ϕ, introduce the $U(1)$ gauge fields A_μ of electromagnetism, choose a renormalized $SU(3)$ gauge theory of real scalar fields, and allow the gauge to be spontaneously broken (Kibble, 1967). Therefore, the Lagrangian is

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}\mathbf{G}^a_{\mu\nu}\cdot\mathbf{G}^{a\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu\psi + \tfrac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi) + \tfrac{1}{2}m^2\phi^\dagger\phi - \tfrac{1}{4}f^2(\phi^\dagger\phi)^2 - \alpha_Y\bar{\psi}\phi\phi^\dagger\psi \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2}$$

$$\mathbf{G}^a_{\mu\nu} = \partial_\mu\mathbf{G}^a_\nu - \partial_\nu\mathbf{G}^a_\mu + g\mathbf{G}^a_\mu\times\mathbf{G}^a_\nu \tag{3}$$

$$D_\mu = \partial_\mu + ig_eA_\mu - \tfrac{1}{2}ig\boldsymbol{\lambda}^a\cdot\mathbf{G}^a_\mu \tag{4}$$

g_e is the electromagnetic coupling constant and g is the Dirac magnetic charge. A_μ are the photon fields. $\mathbf{G}^a_\mu$ are the eight gluon fields. D_μ is the covariant derivative. The space-time indices are $\mu = 0, 1, 2, 3$. The $\boldsymbol{\lambda}^a$ are the Gell-Mann matrices, and $a = 1, \ldots, 8$. The magnetic monopoles are chosen to be represented by the scalar Higgs fields ϕ_1 and ϕ_2, where $\phi = \phi_1 + i\phi_2$. The ψ are the four-component Dirac spinors associated with each quark (dyon) field. Substituting equation (4) into equation (1) gives

$$L = L_1 + L_2 + L_3 \tag{5}$$

with

$$L_1 = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{6}$$

$$L_2 = -\tfrac{1}{4}\mathbf{G}^a_{\mu\nu}\cdot\mathbf{G}^{a\mu\nu} \tag{7}$$

$$\begin{aligned} L_3 = {} & i\bar{\psi}\gamma^\mu\partial_\mu\psi - J^\mu A_\mu + \tfrac{1}{2}[(\partial_\mu\phi^\dagger)(\partial^\mu\phi) + \mathbf{K}^{a\mu}\cdot\mathbf{G}^a_\mu \\ & + \mathbf{C}^{a\mu}\cdot\mathbf{G}^a_\mu + g_e^2\phi^\dagger\phi A_\mu A^\mu + \tfrac{1}{4}g^2\phi^\dagger(\boldsymbol{\lambda}^a\cdot\mathbf{G}^a_\mu)^\dagger(\boldsymbol{\lambda}^a\cdot\mathbf{G}^{a\mu})\phi] \\ & + \tfrac{1}{2}m^2\phi^\dagger\phi - \tfrac{1}{4}f^2(\phi^\dagger\phi)^2 - \alpha_Y\bar{\psi}\phi\phi^\dagger\psi \end{aligned} \tag{8}$$

We will restrict our attention to equation (8), which connects the observed currents to the photon and gluon fields. In equation (8) we have

$$J^{\mu} = g_e \bar{\psi}\gamma^{\mu}\psi + ig_e[\phi^{\dagger}(\partial^{\mu}\phi) - (\partial_{\mu}\phi^{\dagger})\phi] \tag{9}$$

$$\mathbf{K}^{a\mu} = \tfrac{1}{2}g\bar{\psi}\gamma^{\mu}\boldsymbol{\lambda}^{a}\psi + \tfrac{1}{2}g[\phi^{\dagger}\boldsymbol{\lambda}^{a}\partial^{\mu}\phi - (\partial_{\mu}\phi^{\dagger})\boldsymbol{\lambda}^{a}\phi] \tag{10}$$

$$\mathbf{C}^{a\mu} = -\tfrac{1}{2}g_e g[(\phi\boldsymbol{\lambda}^{a})^{\dagger}\phi + \phi^{\dagger}\boldsymbol{\lambda}^{a}\phi]A_{\mu} \tag{11}$$

J^{μ} represents the Maxwell electric current density and a scalar electric current density. From equations (10) and (11), we have eight color magnetic currents $\mathbf{K}^{a\mu}$ and eight color dyonic currents $\mathbf{C}^{a\mu}$.

We now choose a suitable gauge for the two real scalar fields ϕ_1 and ϕ_2, each a triplet of isovector fields, to introduce a symmetry-breaking mechanism into equation (8). In our model, the fields ϕ_1 and ϕ_2 represent spin-0, isospin-1 magnetic monopole and antimonopole in order to preserve the conservation of magnetic charge, such that the magnetic charge is related to the third components of isospin: $Q_m = I_3$.

The potential energy of the Lagrangian (8) will be minimized by choosing the renormalized triplet of isovector fields:

$$\phi_1 = \begin{pmatrix} \phi^{+} \\ \phi^{0} \\ \phi^{-} \end{pmatrix} = \begin{pmatrix} \eta + m/f\sqrt{2} \\ 0 \\ 0 \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} \phi^{-} \\ \phi^{0} \\ \phi^{+} \end{pmatrix} = \begin{pmatrix} \xi + m/f\sqrt{2} \\ 0 \\ 0 \end{pmatrix} \tag{12}$$

where the monopole–antimonopole are represented by the fields η and ξ. Neglecting the spinor terms, we are interested in rewriting the Lagrangian (8) in terms of these new fields:

$$\begin{aligned} &\frac{1}{2}(\partial_{\mu}\phi^{\dagger})(\partial^{\mu}\phi) + \frac{1}{2}m^2\phi^{\dagger}\phi - \frac{1}{4}f^2(\phi^{\dagger}\phi)^2 \\ &\quad = \frac{1}{2}(\partial_{\mu}\eta)(\partial^{\mu}\eta) - \frac{1}{2}m^2\eta^2 + \frac{1}{2}(\partial_{\mu}\xi)(\partial^{\mu}\xi) - \frac{1}{2}m^2\xi^2 \\ &\qquad - \frac{1}{4}f^2(\eta^4 + 2\eta^2\xi^2 + \xi^4) - \frac{mf}{\sqrt{2}}(\eta^3 + \eta^2\xi + \eta\xi^2 + \xi^3) - m^2\eta\xi - \frac{1}{4}\frac{m^4}{f^2} \end{aligned} \tag{13}$$

In equation (13) the three terms have become a massive scalar Higgs particle η and a massive Goldstone boson ξ, each with Dirac mass m of the magnetic monopole. Neglecting the spinor terms, we are interested in the remaining terms of the Lagrangian (8):

$$-\frac{1}{2}J^\mu A_\mu = -g_e\left[(\partial_\mu\eta)\xi - (\partial_\mu\xi)\eta + \frac{m}{f\sqrt{2}}\partial_\mu\eta - \frac{m}{f\sqrt{2}}\partial_\mu\xi\right]A_\mu \tag{14}$$

$$\frac{1}{2}\mathbf{K}^a_\mu\cdot\mathbf{G}^a_\mu = \frac{1}{2}g\left[(\partial_\mu\eta)\left(\xi+\frac{m}{f\sqrt{2}}\right) - (\partial_\mu\xi)\left(\eta+\frac{m}{f\sqrt{2}}\right)\right] \times\left(G^3_\mu + \frac{1}{\sqrt{3}}G^8_\mu\right) \tag{15}$$

$$\frac{1}{2}\mathbf{C}^a_\mu\cdot\mathbf{G}^a_\mu = -\frac{1}{2}g_e g\left(\eta^2+\frac{\sqrt{2}m}{f}\eta+\xi^2+\frac{\sqrt{2}m}{f}\xi+\frac{m^2}{f^2}\right) \times\left(G^3_\mu+\frac{1}{\sqrt{3}}G^8_\mu\right)A_\mu \tag{16}$$

$$\frac{1}{2}g_e^2\phi^\dagger\phi A_\mu A^\mu = \frac{1}{2}g_e^2\left(\eta^2+\frac{\sqrt{2}m}{f}\eta+\xi^2+\frac{\sqrt{2}m}{f}\xi+\frac{m^2}{f^2}\right)A_\mu A^\mu \tag{17}$$

$$\frac{1}{8}g^2\phi^\dagger(\boldsymbol{\lambda}^a\cdot\mathbf{G}^a_\mu)^\dagger(\boldsymbol{\lambda}^a\cdot\mathbf{G}^a_\mu)\phi = \frac{1}{8}g^2\left(\eta^2+\frac{\sqrt{2}m}{f}\eta+\xi^2+\frac{\sqrt{2}m}{f}\xi+\frac{m^2}{f^2}\right) \times\left[(G^1_\mu)^2+(G^2_\mu)^2+(G^3_\mu)^2+(G^4_\mu)^2+(G^5_\mu)^2+\frac{1}{3}(G^8_\mu)^2+\frac{2}{\sqrt{3}}G^3_\mu G^8_\mu\right] \tag{18}$$

$$-\alpha_Y\bar{\psi}\phi\phi^\dagger\psi = -\alpha_Y\left(\eta^2+\frac{\sqrt{2}m}{f}\eta+\xi^2+\frac{\sqrt{2}m}{f}\xi+\frac{m^2}{f^2}\right)\bar{\psi}_r\psi_r \tag{19}$$

In equations (16)–(18), the electric, magnetic, and dyonic currents are recombined to give the coupling between the electromagnetic and strong interactions:

$$\frac{1}{2}\left(\eta^2+\frac{\sqrt{2}m}{f}\eta+\xi^2+\frac{\sqrt{2}m}{f}\xi+\frac{m^2}{f^2}\right)\left[g_e A_\mu - \frac{1}{2}g\left(G^3_\mu+\frac{1}{\sqrt{3}}G^8_\mu\right)\right]^2 \tag{20}$$

We see immediately that the charged spin-1 field is

$$W_\mu = \frac{1}{\sqrt{2}}(A^1_\mu + iA^2_\mu) \tag{21}$$

and has a superheavy mass

$$M_W = g_e\frac{m}{f} \tag{22}$$

M_W is a superheavy vector boson analogous to the electroweak $W^{\pm}$ boson. From equation (18) we have the gluon interactions

$$\frac{1}{8}g^2\frac{m^2}{f^2}[(G_\mu^1)^2+(G_\mu^2)^2+(G_\mu^4)^2+(G_\mu^5)^2]$$

with superheavy charged vector bosons given by

$$X_\mu=\frac{1}{\sqrt{2}}(G_\mu^1+iG_\mu^2), \qquad Y_\mu=\frac{1}{\sqrt{2}}(G_\mu^4+iG_\mu^5) \tag{23}$$

and with masses given by

$$M_X=M_Y=\frac{g}{2}\frac{m}{f} \tag{24}$$

From equation (20) there are neutral spin-1 fields

$$Z_\mu=\left(g_e^2+\frac{g^2}{4}\right)^{-1/2}\left[g_eA_\mu^3-\frac{1}{2}g\left(G_\mu^3+\frac{1}{\sqrt{3}}G_\mu^8\right)\right] \tag{25}$$

and

$$A'_\mu=\left(g_e^2+\frac{g^2}{4}\right)^{-1/2}\left[\frac{1}{2}gA_\mu^3+g_e\left(G_\mu^3+\frac{1}{\sqrt{3}}G_\mu^8\right)\right] \tag{26}$$

Their masses are

$$M_Z=\frac{m}{f}\left(g_e^2+\frac{g^2}{4}\right)^{1/2} \tag{27}$$

$$M_{A'}=0 \tag{28}$$

A'_μ is to be identified as the photon field.

The masses of the superheavy bosons have been studied in the high-energy (>300 GeV) limit by Akers (1992*b*). We now consider the low-energy region (<20 GeV), where we expect unification of the $SU(3)$ magnetic monopoles with the $SU(2)$ results of Benjwal and Joshi (1987).

In the low-energy region, the boson mass formulas scale as

$$\frac{m}{2f}\to m_e \tag{29}$$

and

$$(g_e^2+g^2)^{1/2}\sim g=\frac{137}{2}ne \tag{30}$$

where n is a positive integer ($n=0, 1, \ldots$) and where the Dirac quantization conditions (Dirac, 1931) is introduced into the right side of equation (30). Substituting equations (29) and (30) into equation (27), we find the boson masses

$$M_n = \frac{g}{e} m_e = \frac{137}{2} n m_e \tag{31}$$

in units of MeV. The right side of equation (31) is recognized as Nambu's empirical mass formula (Nambu, 1952; Hokkyo, 1987). Thus, the existence of a Dirac unit of magnetic charge leads to the well-known Nambu formula for particle masses.

3. DYON SOLUTIONS IN *SU*(2) GROUP SPACE

Benjwal and Joshi (1987) studied the dyon solutions of a non-Abelian field tensor in a Lagrangian with spontaneous breaking of $SU(2)$ symmetry by a triplet of Higgs fields. We consider the solutions discovered by Benjwal and Joshi to prove that their vector bosons also obey the Nambu formula for particle masses.

Benjwal and Joshi derived the Lagrangian density of a dyon in the $SU(2)$ gauge group after a transformation of the scalar Higgs fields $\phi \to \phi' = U\phi$. The Lagrangian density may be given by

$$L_0 = \tfrac{1}{2}[D_\mu \phi)^a (D^\mu \phi)^a + (D'_\mu \phi)^a (D'^\mu \phi)^a] - V(\phi^a) - \tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} \tag{32}$$

where

$$(D_\mu \phi)^a = (\partial_\mu \delta^{ac} + e\varepsilon^{abc} A^b_\mu)\phi^c \tag{33}$$

$$(D'_\mu \phi)^a = (\partial_\mu \delta^{ac} + g\varepsilon^{abc} B^b_\mu)\phi^c \tag{34}$$

and

$$V(\phi^a) = \frac{\lambda}{4} (\phi^a \phi^a - f_1^2)^2 \tag{35}$$

By choosing the unit vector $\mathbf{n}$ along the third direction in $SU(2)$ group space, Benjwal and Joshi discovered that the two components A^1_μ, A^2_μ and B^1_μ, B^2_μ acquire the masses

$$m_e = ef_1 \tag{36}$$

and

$$m_g = gf_1 \tag{37}$$

The third components A^3_μ and B^3_μ remain massless. Thus, the spectrum of spontaneously broken $SU(2)$ theory contains two vector bosons of charges $\pm e$, $\pm g$ with the masses given in equations (36) and (37). Benjwal and Joshi continued their study into the temporal gauge without noticing that

f_1 can be eliminated between equations (36) and (37) to give

$$M_g = \frac{g}{e} m_e = \frac{137}{2} n m_e$$

which is the Nambu empirical mass formula (31).

4. MACGREGOR'S CONSTITUENT-QUARK MODEL

We now consider the experimental data (Particle Data Group, 1988) to show that the Nambu empirical mass formula (31) has a basis for its existence in the particle spectrum. We note that the Nambu formula yields the mass series 0, 35, 70, 105, 140, ... in units of MeV. This mass series is tabulated by Nambu (1952), and the 70-MeV boson is the famous mass quantum of MacGregor (1990).

MacGregor developed a constituent-quark model of elementary particles, which are related to the masses of the electron, muon, and pion. A comparison of Nambu's formula with MacGregor's model is shown in Table I. The Nambu empirical mass formula, which is based upon the existence of magnetic charge, is equivalent to the constituent-quark model of MacGregor.

In Table I, the notation is from MacGregor (1990). The mass series reproduces exactly the elementary particle spectrum as shown in Tables II and III when the binding energies are taken into an account (MacGregor, 1990). Meson resonances are shown in Table II up to 1200 MeV. For additional resonances above 1200 MeV, see the work by MacGregor (1990).

Thus, we conclude that the 70-MeV quantum of MacGregor has its origin in the existence of Dirac magnetic charges.

5. THE EMPIRICAL MASS FORMULA $m \approx N(3m_e)$

Frosch (1991) recently proved the existence of an empirical mass formula $m \sim N(3m_e)$ for leptons and hadrons. Frosch's evidence for a "rest mass quantum" of $3m_e$ was derived from among 47 elementary particles as shown in Table IV. The evidence was based upon a study of the deviation from an exact quantum law given by

$$x_i = \frac{m_i}{3m_e} - N_i \tag{38}$$

The results for different weighting functions $D(M_0)$ and $D'(M_0)$ are shown in Fig. 1. These weighting functions have pronounced minima close to $3m_e$. Frosch also studied random functions for mean deviations from the

Table I. Comparison of Nambu's Empirical Mass Formula with MacGregor's Constituent-Quark Model[a]

n	m_n	(MeV)	Notation from MacGregor's constituent-quark model	Elementary particle mass
0	$\frac{0}{2}m$	0		
1	$\frac{1}{2}m$	35		
2	$\frac{2}{2}m$	70	m (70-MeV quantum)	
3	$\frac{3}{2}m$	105	μ	μ
4	$\frac{4}{2}m$	140	B (boson excitation)	π
5	$\frac{5}{2}m$	175		
6	$\frac{6}{2}m$	210	F (fermion excitation)	2μ
7	$\frac{7}{2}m$	245		
8	$\frac{8}{2}m$	280	$BB = F + m$	2π
9	$\frac{9}{2}m$	315	u quark	3μ
10	$\frac{10}{2}m$	350		
11	$\frac{11}{2}m$	385	u' quark ($u' = u + m$)	
12	$\frac{12}{2}m$	420	X (excitation quantum) $X = FF = 3B$	$3\pi = 4\mu$
13	$\frac{13}{2}m$	455		
14	$\frac{14}{2}m$	490		
15	$\frac{15}{2}m$	525	s quark	5μ
16	$\frac{16}{2}m$	560	BX	4π
17	$\frac{17}{2}m$	595	s' quark ($s' = s + m$)	
18	$\frac{18}{2}m$	630	$FFF = XF$	6μ

Table I. Continued.

n	m_n	(MeV)	Notation from MacGregor's constituent-quark model	Elementary particle mass
19	$\frac{19}{2}m$	665		
20	$\frac{20}{2}m$	700	BBX	5π
21	$\frac{21}{2}m$	735		7μ
22	$\frac{22}{2}m$	770	BFX	
23	$\frac{23}{2}m$	805		
24	$\frac{24}{2}m$	840	$FFFF = XX$	$6\pi = 8\mu$
25	$\frac{25}{2}m$	875		
26	$\frac{26}{2}m$	910		
27	$\frac{27}{2}m$	945	$3u$ (nucleon quarks)	n, p
28	$\frac{28}{2}m$	980	BXX	7π
29	$\frac{29}{2}m$	1015		
30	$\frac{30}{2}m$	1050	$FFFFF = XXF$	
...				

[a] $m_n = (g/e)m_e = (n/2\alpha)m_e = (n/2)(137)m_e$, where n is a positive integer.

formula (38). Frosch concludes, "It may be worth emphasizing that the $3m_e$ formula is empirical in the strict sense; i.e., we do not know of any theoretical ideas that suggest such a 'quantum' law" (Frosch, 1991).

We note that for $n = 3$ in Table I the muon is the first elementary particle derived from Nambu's empirical mass formula and from MacGregor's constituent-quark model. Frosch's mass formula is a higher-order set which contains the Nambu mass formula. From equation (31) we have ($n = 3$)

$$M_3 = (137/2)3m_e = 68.5(3m_e) = \mu \tag{39}$$

Table II. Constituent-Quark Assignments for All of the Observed $J = 0$ Meson and Kaon Resonances up to 1200 MeV[a]

CQ excitation	Particle	CQ mass (MeV)	Experimental mass (MeV)	BE_{CQ}
Spin-zero nonstrange resonances				
$B^{\pm} = m_u \bar{m}_d$	$\pi^{\pm}$	145	140	3.4%
$B^0 = m_u \bar{m}_u$	π^0	140	135	3.6%
$B^0 X_B$	η	560	549	2.0%
$B^0 X_B X_B$	η'	980	958	2.2%
$B^0 X_B X_F$	f_0	980	976	0.4%
$B^{\pm} X_B X_F$	a_0	985	983	0.2%
Spin-zero strange resonances				
$m_u X_B$	K^+	490	494	0%
$m_d X_B$	K_L^0	495	498	0%
$m_d X_F$	K_S^0	495	498	0%

[a]The CQ notation, masses, and binding energies are from MacGregor (1990). With the mass assignments $m_u = 70$ MeV, $m_d = 74.6$ MeV, $F = 210$ MeV, and $X = 420$ MeV, the CQ configurations shown yield reasonable mass values.

Frosch did not take into an account the elementary particle binding energies, as MacGregor (1990) has shown. Thus, $N = 69$ in Table IV showing Frosch's calculations is actually 68.5 as shown in equation (39). Frosch independently and unknowingly discovered evidence for the Dirac unit of magnetic charge $g = (137/2)e = 68.5e$.

Table III. Constituent-Quark Assignments for the Baryon and Hyperon Ground States[a]

CQ excitation mechanism	CQ state	CQ mass (MeV)	Experimental mass (MeV)	BE_{CQ}
$FXXXX \Rightarrow N\bar{N}$	uud	950	$p^+(938)$	1.3%
	udd	955	$n^0(940)$	1.6%
$\pi BX + N \Rightarrow \Lambda + K$	uds	1160	$\Lambda^0(1116)$	3.9%
$\pi FX + N \Rightarrow \Sigma + K$	uus'	1225	$\Sigma^+(1189)$	3.0%
	uds'	1230	$\Sigma^0(1193)$	3.1%
	dds'	1235	$\Sigma^-(1197)$	3.2%
$\bar{K}X + N \Rightarrow \Xi + K$	uss	1365	$\Xi^0(1315)$	3.8%
	dds	1370	$\Xi^-(1321)$	3.7%
$\bar{K}XXX + N \Rightarrow \Omega + K + K$	$ss's'$	1715	$\Omega^-(1672)$	2.5%

[a]The CQ notation, masses, and binding energies are from MacGregor (1990). With the mass assignments $u(315)$, $d(320)$, $s(525)$, and $s'(595)$, the CQ configurations shown yield reasonable mass values.

Table IV. Experimental Values of Rest Mass m_i, Integer N_i, and Deviation x_i from Calculations by Frosch (1991)

Particle	Experimental mass m_i (MeV)	N_i	x_i
$\mu^\pm$	105.658387 ± 0.000034	69	-0.077246 ± 0.000022
π^0	134.9739 ± 0.0006	88	$+0.04576 \pm 0.00039$
$\pi^\pm$	139.5675 ± 0.0004	91	$+0.04224 \pm 0.00026$
$K^\pm$	493.646 ± 0.009	322	$+0.0136 \pm 0.0059$
K^0	497.671 ± 0.031	325	-0.361 ± 0.020
η	548.8 ± 0.6	358	-0.01 ± 0.39
$\rho^\pm$	768.3 ± 0.5	501	$+0.18 \pm 0.33$
ρ^0	768.57 ± 0.62	501	$+0.35 \pm 0.40$
ω	781.95 ± 0.14	510	$+0.079 \pm 0.091$
$K^{*\pm}$	891.83 ± 0.24	582	-0.24 ± 0.16
K^{*0}	896.10 ± 0.28	585	-0.46 ± 0.18
η'	957.50 ± 0.24	625	-0.41 ± 0.16
ϕ	1019.412 ± 0.008	665	-0.0203 ± 0.0052
D^0	1864.5 ± 0.5	1216	$+0.24 \pm 0.33$
$D^\pm$	1869.3 ± 0.4	1219	$+0.38 \pm 0.26$
D_s	1968.8 ± 0.7	1284	$+0.28 \pm 0.46$
$D^{*\pm}$	2010.1 ± 0.6	1311	$+0.22 \pm 0.39$
D_{s1}	2536.5 ± 0.8	1655	-0.40 ± 0.52
$J/\psi(1S)$	3096.93 ± 0.09	2020	$+0.180 \pm 0.059$
$\chi_{c0}(1P)$	3415.1 ± 1.0	2228	-0.27 ± 0.65
$\chi_{c1}(1P)$	3510.6 ± 0.5	2290	$+0.02 \pm 0.33$
$\chi_{c2}(1P)$	3556.3 ± 0.4	2320	-0.17 ± 0.26
$\psi(2S)$	3686.00 ± 0.10	2404	$+0.440 \pm 0.065$
$\Upsilon(1S)$	9460.32 ± 0.22	6171	-0.13 ± 0.14
$\chi_{b1}(1P)$	9891.9 ± 0.7	6453	-0.35 ± 0.46
$\chi_{b2}(1P)$	9913.2 ± 0.6	6467	-0.45 ± 0.39
$\Upsilon(2S)$	10023.30 ± 0.31	6538	$+0.37 \pm 0.20$
$\chi_{b1}(2P)$	10255.2 ± 0.4	6690	-0.36 ± 0.26
$\chi_{b2}(2P)$	10269.0 ± 0.7	6699	-0.36 ± 0.46
$\Upsilon(3S)$	10355.3 ± 0.5	6755	-0.06 ± 0.33
p	938.27231 ± 0.00028	612	$+0.05090 \pm 0.00018$
n	939.56563 ± 0.00028	613	-0.10544 ± 0.00018
Λ	1115.63 ± 0.05	728	-0.256 ± 0.033
Σ^+	1189.37 ± 0.07	776	-0.154 ± 0.046
Σ^0	1192.55 ± 0.10	778	-0.079 ± 0.065
Σ^-	1197.43 ± 0.06	781	$+0.104 \pm 0.039$
Δ^{++}	1231.1 ± 0.2	803	$+0.07 \pm 0.13$
Δ^0	1233.8 ± 0.2	805	-0.17 ± 0.13
Ξ^0	1314.9 ± 0.6	858	-0.27 ± 0.39
Ξ^-	1321.32 ± 0.13	862	-0.081 ± 0.085
Σ^{*+}	1382.8 ± 0.4	902	$+0.02 \pm 0.26$
Σ^{*0}	1383.7 ± 1.0	903	-0.39 ± 0.65
Σ^{*-}	1387.2 ± 0.5	905	-0.11 ± 0.33
Λ^*	1519.5 ± 1.0	991	$+0.20 \pm 0.65$
Ξ^{*0}	1531.80 ± 0.32	999	$+0.22 \pm 0.21$
Ξ^{*-}	1535.0 ± 0.6	1001	$+0.31 \pm 0.39$
Ω^-	1672.43 ± 0.32	1091	-0.05 ± 0.21

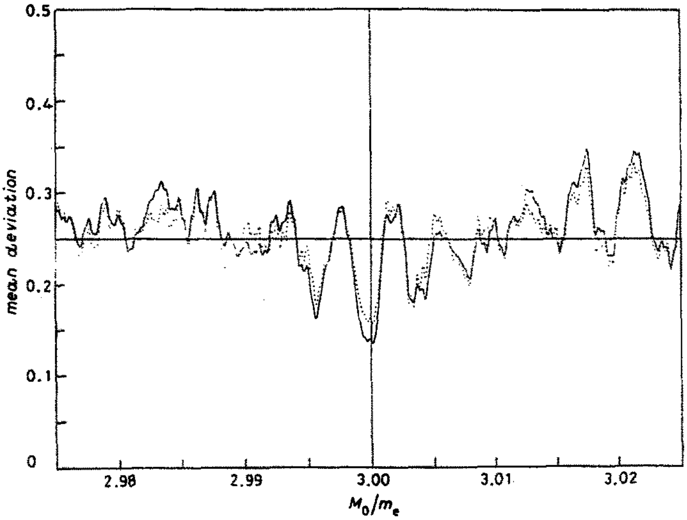

Fig. 1. Mean deviation of the 47 particle masses in Table IV from integral multiples of M_0 near $M_0 = 3m_e$. The calculations are taken from Frosch (1991). The solid and dotted curves represent the function $D(M_0)$ and $D'(M_0)$, respectively. Note that these functions have pronounced minima close to $3m_e$.

6. CONCLUSION

We summarize the evidence for the discovery of quarks carrying the Dirac unit of magnetic charge $g = (137/2)e$. The presence of Zeeman splitting in meson states is well known (Akers, 1985). Akers (1986) presented an analysis of the Zeeman splitting and found evidence for quarks with magnetic sources to account for the meson spectrum. Based upon the dyonium model (Schwinger, 1969), Akers (1987) presented a model of the Paschen–Back effect in dyonium and predicted the existence of a new η meson at 1814 MeV. The experimental evidence for dyonic quarks in terms of this η resonance has been confirmed by Bisello *et al.* (1989). Akers (1992*a*) recently presented the experimental data at the Vancouver Meeting of the American Physical Society. Furthermore, Akers (1992*c*) has discovered a relationship between the Dirac monopole and elementary particle lifetimes.

In this paper, we have not considered the problem of fractional electric charges and monopoles. Other authors have recently studied the coexistence of fractional electric charges and monopoles (Rana and Rajput, 1993; Zhang, 1987). Likewise, we have not considered the problem of elementary particle spins and statistics. Li (1987) may have a handle on this difficult problem.

However, we have studied the role of Dirac magnetic monopoles in the origin of mass. Within the framework of the theory of the strong QCD and electromagnetic interactions, magnetic monopoles may be treated as Goldstone and Higgs bosons to yield the Nambu empirical mass formula (Nambu, 1952). The $SU(2)$ model of Benjwal and Joshi (1987) has also as its foundation the Nambu formula, which is based upon the existence of Dirac magnetic charges. By considering the particle mass spectrum, we have shown that the Nambu formula is equivalent to the constituent-quark model of MacGregor (1990). Moreover, Frosch (1991) derived an empirical mass formula which reveals a quantum law with a "rest mass quantum" of $3m_e$. Frosch independently and unknowingly discovered evidence for magnetic charge.

REFERENCES

Akers, D. (1985). In *Proceedings of the Oregon Meeting*, Rudolph C. Hwa, ed., World Scientific, Singapore.

Akers, D. (1986). *International Journal of Theoretical Physics*, **25**, 1281.

Akers, D. (1987). *International Journal of Theoretical Physics*, **26**, 451.

Akers, D. (1992*a*). In *The Vancouver Meeting: Particles & Fields '91*, D. Axen, D. Bryman, and M. Comyn, eds., World Scientific, Singapore, p. 931.

Akers, D. (1992*b*). Dirac magnetic monopoles as Goldstone and Higgs bosons in the origin of mass, preprint.

Akers, D. (1992*c*). *Nuovo Cimento*, **105A**, 935.

Benjwal, M. P., and Joshi, D. C. (1987). *Physical Review D*, **36**, 629.

Bisello, D., *et al.* (1989). *Physical Review D*, **39**, 701.

Chang, C. (1972). *Physical Review D*, **5**, 950.

Dirac, P. A. M. (1931). *Proceedings of the Royal Society of London A*, **133**, 60.

Frosch, R. (1991). *Nuovo Cimento*, **104A**, 913.

Hokkyo, N. (1987). *Nuovo Cimento*, **98A**, 787.

Kibble, T. W. B. (1967). *Physical Review*, **155**, 1554.

Li, Hua-Zhong. (1987). *Physical Review D*, **35**, 2615.

MacGregor, M. H. (1990). *Nuovo Cimento*, **103A**, 983.

Mandelstam, S. (1975). *Physics Letters*, **53B**, 476.

Nambu, Y. (1952). *Progress in Theoretical Physics*, **7**, 595.

Nambu, Y. (1974). *Physical Review*, **10**, 4262.

Particle Data Group (1988). *Physics Letters B*, **204**, 1.

Polyakov, A. (1974). *JETP Letters*, **20**, 194.

Rana, J. M. S., and Rajput, B. S. (1993). *International Journal of Theoretical Physics*, **32**, 357.

Schwinger, J. (1969). *Science*, **165**, 757.

'T Hooft, G. (1974). *Nuclear Physics B*, **79**, 276.

Zhang, Huazhong (1987). *Physical Review D*, **36**, 1868.